

STRATEGIC ASSET ALLOCATION AND RISK ATTRIBUTION

Philippe Grégoire : PhD, MD Orfival, Professor of finance, Louvain School of Management, University of Louvain.

philippe.gregoire@orfival.com

Orfival

Av Lenoir 2A

1348 Louvain-la-Neuve

Belgium

Philippe Vandooren : CEO Orfival

philippe.vandooren@orfival.com

Orfival

Av Lenoir 2A

1348 Louvain-la-Neuve

Belgium

In this article, we propose a model for risk attribution that explains the difference between the risk of a portfolio and its strategic asset allocation. A first version of the risk attribution model explains the changes in the risk profile by allocation, diversification and selection decisions. The second version is more intuitive as it attributes the risk, first to tactical allocation decisions and, second to the stock picking. The changes in the correlations or the contribution to the overall risk of the portfolio are measured by a term called 'diversification effect'. Risk attribution contributes to a better understanding of the sources of performance as it links up the source of active returns to active risk.

This article aims to provide a framework that explains the contribution of active management decisions to the actual level of risk. The model assumes that the fund has its own strategic asset allocation represented by different indexes and that the manager is allowed to decide a tactical asset allocation and to select specific stocks. The manager is active in allocation and selection to capture excess returns by adopting different risk profiles. In this sense, the model differs from other risk attribution models which explain the level of tracking error caused by allocation and selection

decisions (Menchero and Hu, 2006, Philippe Bertrand, 2008). The latter are suitable for funds that are managed against a benchmark, or a peer group, while the former are suitable for portfolios that have a strategic asset allocation. Given this assumption of a top-down investment process, the model will attribute the difference of risk to tactical asset allocation, selection and contribution to the level of diversification. In the field of return attribution (Brinson et al., 1986; Karnoski and Singer, 1994), practitioners often agree that the most important step in selecting an attribution model is to identify the active investment decision process. Choosing a Brinson style model is adequate for “Top-Down” strategies based on allocation among asset classes or industries and stock picking. This principle must hold for risk attribution; it is the investment process that defines the type of models that must be used. The model focuses on top-down processes as we assume that the fund has its own strategic allocation represented by a set of indexes. The fund manager is active by deciding on tactical asset allocation and by selecting specific stocks. From a risk perspective, the allocation decisions have an impact on the correlation structure of the portfolio and the amount of risk allocated to each class. The selectivity changes the level of volatility in each class. These three effects, allocation, correlation and selection should be emphasized in risk attribution models. To illustrate the need of a model, let us take the case of a pension fund that has defined a strategic asset allocation based on an ALM model. The asset allocation committee of the pension fund decides to deviate from strategic allocation and selects specific stocks, attempting to increase the expected rate of return. Although it is important to measure the contributions to return that arise from tactical allocation and stock picking, it is also valuable to assess whether this is achieved in an efficient way- i.e. is the excess return associated with active decisions large enough to cover the excess of risk? Risk attribution contributes to a better estimate of the risk-return trade-off of active management as it attributes the excess risk to allocation and selection decisions.

The aim of the model is to explain which part of the excess risk comes from the changes to the correlation structure, the allocation decisions and the selectivity in each asset class¹. We also present the model from a more intuitive view that explains the variations in risk by successive portfolios, i.e. from the strategic portfolio to the tactical portfolio and from the tactical portfolio to the managed portfolio.

The first section presents the model that is based on the risk decomposition proposed by Menchero and Hu (2006) and Grégoire and Van Oppens (2006). The second section analyzes the model when trading is allowed and the third section gives a more intuitive interpretation of the model. An example of this is presented in section 4.

Risk attribution model

The total risk of the portfolio and the benchmark (strategic asset allocation) are measured by the standard deviation,

$$\sigma_P = \frac{1}{\sigma_P} \sum_{i,j=1}^N w_i \sigma_{ij} w_j$$

Using the marginal contribution to standard deviation,

$$\frac{\partial \sigma_P}{\partial w_j} = \frac{1}{\sigma_P} \sum_{i,j=1}^N \sigma_{ij} w_j = \rho_{iP} \sigma_i$$

It is possible to express the total risk as the weighted sum of marginal contributions

$$\sigma_P = \sum_{i=1}^N w_i \rho_{iP} \sigma_i \tag{Eq. 1}$$

Equation 1 shows that the total risk of the portfolio is a function of 1°, the correlation structure ρ_{iP} , 2°, the amount w_i allocated to each class and 3°, the risk σ_i of each class. In the case of a fund that has a strategic allocation, the total risk of the actual portfolio is the result of first; tactical asset allocation that modify the correlation structure ρ_{iP} and the amount w_i invested in each class, and second; selection decisions that have a direct impact of the level of risk σ_i . In the model, the impact of changes in correlation that results from tactical decisions is called the ‘diversification effect’ and the consequence of the different amounts of risk that are allocated to each class is named ‘allocation

¹ We refer to asset class, but the model apply to any segmentation that defines the allocation decision

effect'. The latter measures the contribution to the excess risk of overweighting (underweighting) each class. Finally, the excess risk associated to stock picking activities is called the selection effect. Using equation 1, we can write

$$\sigma_P = \sum_{i=1}^N \underbrace{w_i}_{\text{Allocation}} \times \underbrace{\rho_i}_{\text{Diversification}} \times \underbrace{\sigma_i}_{\text{Selection}} \quad \text{for the managed portfolio and}$$

$$\underline{\sigma}_B = \sum_{i=1}^N \underbrace{\underline{w}_i}_{\text{Allocation}} \times \underbrace{\underline{\rho}_i}_{\text{Diversification}} \times \underbrace{\underline{\sigma}_i}_{\text{selection}} \quad \text{for the associated benchmark or the strategic asset allocation}^2.$$

The expression of the volatility for both the portfolio and the strategic asset allocation allows us to explain the difference of volatility or excess risk as

$$\sigma_P - \underline{\sigma}_B = \sum_{i=1}^N \underbrace{(w_i - \underline{w}_i) \times \underline{\rho}_i \times \underline{\sigma}_i}_{\text{Allocation effect}} + \sum_{i=1}^N \underbrace{w_i \times (\rho_i - \underline{\rho}_i) \times \underline{\sigma}_i}_{\text{diversification effect}} + \sum_{i=1}^N \underbrace{w_i \rho_i \times (\sigma_i - \underline{\sigma}_i)}_{\text{selection effect}} \quad \text{Eq. 2}$$

Each term of equation 2 has the same structure as the contribution to risk (equation 1), i.e. a weight times a correlation times a volatility. The allocation effect measures the consequences of investing the active weights in a portfolio that displays correlations and volatilities similar to the benchmark. The diversification effect shows the result of the changes in correlations that come from setting the tactical portfolio. The consequence of the changes of volatility in each class that arise from stock picking is given by the term called “selection effect”. The attribution model³ explains the change in the risk profile by the consequences of 1° active weights or tactical allocation ($w_i - \underline{w}_i$), 2° selectivity ($\sigma_i - \underline{\sigma}_i$) and 3° correlation ($\rho_i - \underline{\rho}_i$).

To illustrate the model, let's assume a fund that is equally invested in each class (25%), that replicates exactly the strategic asset allocation and that allows for stock picking. As illustrated by Table -1-, the total return of this fund is 11.23%, which is the return realized by a portfolio that is rebalanced on a monthly basis to keep weights constant⁴. Although we have considered that the allocation is equal to the benchmark, we must keep in mind that the stock picking decisions have an effect on the volatility in each class and on the correlation structure. The correlations between classes in the portfolio are slightly different from the correlations in the benchmark (see table 1).

² We underline the characteristics of the benchmark or strategic asset allocation. A correlation that is underline means that it is the correlation of the asset class within the benchmark or strategic asset allocation.

³ The proposed risk attribution model differs from the model of Menchero (2006) in the sense that it aims at decomposing the excess volatility (not the tracking error).

⁴ This explains why the return is not equal to the weighted sum of return as it is calculated by the time weighted return of a portfolio with monthly rebalancing

This explains why the diversification effect is not zero. The total increase in volatility comes mainly from “Large growth” and “Small growth”. The selectivity in “Large Growth” has generated a negative excess return, has increased the correlation with the managed portfolio and the volatility of this class. As the correlation of “Large Growth” with the portfolio has increased to 0.56 (from 0.43), the contribution of stock picking to diversification is $0.30\% = 0.25 \times (0.560 - 0.43) \times 9.15\%$. The selection effect is equal to $0.44\% = 0.25 \times 0.56 \times (12.26 - 0.9.15)$. The same calculations are made for each class so that the attribution model explains the excess risk of 2.86% as the consequences of the new correlations, 0.64%, and the increase of volatility in each class, 2.22%. Obviously, the allocation effect is zero.

	Fund Ret	Bench Ret	Correl Ptf	Correl Bench	Vol Ptf	Vol Bench	Alloc	Diversif	select
Large growth	6.43%	9.25%	0.56	0.43	12.26%	9.15%	0.00%	0.30%	0.44%
Small value	13.48%	8.25%	0.52	0.58	17.40%	11.50%	0.00%	-0.18%	0.77%
Small growth	9.03%	12.94%	0.73	0.72	21.36%	15.61%	0.00%	0.04%	1.05%
Large value	10.55%	7.37%	0.38	0.18	9.31%	9.59%	0.00%	0.48%	-0.03%
Total	11.23%	10.27%	1	1	8.74%	5.88%	0.00%	0.64%	2.22%

Table -1-

In the second example presented in table -2-, we assume that the fund decides a tactical asset allocation though stock picking is the same as in table -1-. Tactical asset allocation has contributed to increase the volatility of the fund by 1% (to 9.74%). This higher risk is mainly explained by the decision to overweight “Small Growth”, which contributes to 1.12% to the allocation effect, and by the stock picking, which accounts for 1.57% in the selection effect (see table -2-). Comparing the results with table -1-, we observe that tactical allocation decisions have barely a small effect on the degree of diversification (0.57% vs. 0.64%) and on the contribution of stock picking (2.44% vs. 2.22%). However, the tactical asset allocation increases the risk because larger amounts are allocated to asset classes that have a higher risk (for example, allocation to “Small Growth” increases the risk by 1.12%).

	Fund %	Bench %	Correl Ptf	Correl Bench	Vol Ptf	Vol Bench	Alloc	Diversif	select
Large growth	20%	25%	0.48	0.43	12.26%	9.15%	-0.20%	0.09%	0.30%
Small value	25%	25%	0.40	0.58	17.40%	11.50%	0.00%	-0.51%	0.60%
Small growth	35%	25%	0.78	0.72	21.36%	15.61%	1.12%	0.34%	1.57%
Large value	20%	25%	0.52	0.18	9.31%	9.59%	-0.09%	0.65%	-0.03%
Total			1	1	9.74%	5.88%	0.84%	0.57%	2.44%

Table -2-

Although equation 2 gives valuable highlights on the sources of risk, it does not apply to funds that take several tactical allocation decisions during the reporting period. In the next section, we generalize the model to allow for trading or weights that change over time.

Attribution with trading

To generalize the model to a multi-periodic framework with trading or time varying weights, we use the following result to express the standard deviation as a sum of contributions

$$\sigma(r_P) = \sum_{i=1}^N \rho \left(\underbrace{w_i r_i}_{\text{contribution to return}}, r_P \right) \times \sigma(w_i r_i)$$

where

$$\sigma(r_P) = \sqrt{\frac{1}{T-1} \sum_{t=1}^T (r_{P,t} - \bar{r}_P)^2} \text{ and } \sigma(w_i r_i) = \sqrt{\frac{1}{T-1} \sum_{t=1}^T (w_{i,t} r_{i,t} - \bar{w}_i r_i)^2}$$

$r_{P,t}$ is the return of portfolio P over a time interval $[t-1, t]$ and \bar{r}_P is the average return of portfolio P over T periods.

Using the expression of the volatility as the sum of contributions, it is possible to modify equation 2 to take into account trading and to establish a general framework for risk attribution

$$\begin{aligned} \sigma_P - \sigma_B = & \sum_{i=1}^N \underbrace{\rho(w_i r_i, r_B) \times (\sigma(w_i r_i) - \sigma(w_i r_i))}_{\text{Allocation effect}} + \sum_{i=1}^N \underbrace{(\rho(w_i r_i, r_P) - \rho(w_i r_i, r_B)) \times \sigma(w_i r_i)}_{\text{diversification effect}} \\ & + \sum_{i=1}^N \underbrace{\rho(w_i r_i, r_P) \times (\sigma(w_i r_i) - \sigma(w_i r_i))}_{\text{Selection effect}} \end{aligned} \quad \text{Eq. 3}$$

As in the case of constant weights, the allocation term measures the effect of over(under)weighting different classes. In this expression, as weights change over time, the correlation of the contribution to return with the benchmark is multiplied by the difference of volatility of contributions to return, i.e. the cross contribution (portfolio's weights time benchmark return) minus the passive contribution (benchmark's weights time benchmark's return). This first term shows the effect of active weights. The diversification effect measures the consequences of a new correlation structure (correlation in the benchmark minus correlation in the portfolio) assuming tactical allocation and the absence of stock picking. The last term of equation 3 highlights the effect associated to stock picking given the tactical allocation and the correlations. In fact, this term is equal to the portfolio

correlation times the difference between the volatility of the portfolio's contributions to and the cross contribution.

A more intuitive presentation of the model

The model explains how the manager has changed the risk profile by first deciding a tactical allocation and then selecting specific stocks. In other words, the investment decision process is a two step process that is seen as moving from a benchmark to a tactical portfolio and then from a tactical portfolio to the actual portfolio.

Figure 1 gives a representation of the model.

$\underline{w}_i \times \underline{r}_i$ Benchmark	Risk $\sigma(\underline{w}_i \times \underline{r}_i) \Leftrightarrow \sigma(w_i \times \underline{r}_i)$ Benchmark correlations $\rho(\underline{w}_i \times \underline{r}_i; r_B)$ ①	$w_i \times \underline{r}_i$ Tactical Portfolio	Risk $\sigma(w_i \times \underline{r}_i) \Leftrightarrow \sigma(w_i \times r_i)$ Tactical correlations $\rho(w_i \times \underline{r}_i; r_{TP})$ ③	$w_i \times r_i$ Actual Portfolio
	Diversification $\rho(\underline{w}_i \times \underline{r}_i; r_B) \Leftrightarrow \rho(w_i \times \underline{r}_i; r_{TP})$ Tactical risk $\sigma(w_i \times \underline{r}_i)$ ②		Diversification $\rho(w_i \times \underline{r}_i; r_{TP}) \Leftrightarrow \rho(w_i \times r_i; r_P)$ Portfolio risk $\sigma(w_i \times r_i)$ ④	

Figure -1- A successive portfolio model

The tactical portfolio has the same allocation as the actual portfolio but the stock selection is identical to the benchmark. The returns of the benchmark, of the tactical and actual portfolio are respectively the sum of contributions, $\underline{w}_i \times \underline{r}_i$, $w_i \times \underline{r}_i$ and $w_i \times r_i$.

By deciding a specific allocation, i.e. moving from the benchmark to a tactical portfolio, the manager changes the risk profile by allocating more or less money to each asset class. Therefore, the risk budget allocated to each class varies from $\sigma(\underline{w}_i \times \underline{r}_i)$ to $\sigma(w_i \times \underline{r}_i)$ (Box 1) and the level

of diversification is influenced by the new correlations $\rho(w_i \times \underline{r}_i; r_{TP})$ (Box 2). The alteration of risk that results from creating a tactical portfolio is given firstly, by the change in risk allocation leaving the correlations unchanged, i.e. $[\sigma(w_i \times \underline{r}_i) - \sigma(\underline{w}_i \times \underline{r}_i)] \times \rho(\underline{w}_i \times \underline{r}_i; r_B)$ and secondly, by the change in the level of diversification that is a consequence of the new allocation, i.e. $[\rho(w_i \times \underline{r}_i; r_{TP}) - \rho(\underline{w}_i \times \underline{r}_i; r_B)] \times \sigma(w_i \times \underline{r}_i)$. The level of risk associated to the tactical portfolio is

$$\begin{aligned} \sigma(r_{TP}) &= \sum_{i=1}^N \rho(\underline{w}_i r_i; r_{TP}) \times \sigma(w_i \underline{r}_i) \\ &= \sigma(r_B) + \underbrace{\sum_{i=1}^N \rho(\underline{w}_i \underline{r}_i; r_B) \times (\sigma(w_i \underline{r}_i) - \sigma(\underline{w}_i \underline{r}_i))}_{\text{Risk Allocation}} + \underbrace{\sum_{i=1}^N (\rho(w_i \underline{r}_i; r_{TP}) - \rho(\underline{w}_i \underline{r}_i; r_B)) \times \sigma(w_i \underline{r}_i)}_{\text{Diversification}} \end{aligned}$$

Eq 4a

As we observe, the term “Risk Allocation” will be positive (increase in risk) when the manager allocates more weight to an asset class that has a positive correlation with the benchmark. The term “Diversification” shows that the risk rises when the allocation in an asset class increases its correlation with the tactical portfolio.

The second step in the investment decision process is stock picking. The risk that results is explained by the difference of risk between the tactical portfolio and the actual portfolio, i.e. the difference between $\sigma(r_P)$ and $\sigma(r_{TP})$. Stock picking modifies both, the level of risk of each allocation’s class and on the level of diversification. The former is measured by $[\sigma(w_i \times r_i) - \sigma(w_i \times \underline{r}_i)] \times \rho(w_i \times \underline{r}_i; r_{TP})$, which is the difference of risk under the assumption that the level of diversification corresponds to that of the tactical portfolio (box 3), and the latter is given by $[\rho(w_i \times r_i; r_P) - \rho(w_i \times \underline{r}_i; r_{TP})] \times \sigma(w_i \times r_i)$, which measures the consequences of a change in correlations (box 4). The level of risk associated to the actual portfolio is

$$\begin{aligned} \sigma(r_P) &= \sum_{i=1}^N \rho(w_i r_i; r_P) \times \sigma(w_i r_i) \\ &= \sigma(r_{TP}) + \underbrace{\sum_{i=1}^N \rho(w_i \underline{r}_i; r_{TP}) \times (\sigma(w_i r_i) - \sigma(w_i \underline{r}_i))}_{\text{Risk Selection}} + \underbrace{\sum_{i=1}^N (\rho(w_i r_i; r_P) - \rho(w_i \underline{r}_i; r_{TP})) \times \sigma(w_i r_i)}_{\text{Diversification}} \end{aligned}$$

Eq 4b

Equation 3 in the previous section results from the merger of equation 4a and 4b, which demonstrates that the two approaches are equivalent.

Example

To illustrate the model, we use the data of the active strategy presented in table -2-, holding the assumption of constant weights, i.e. rebalancing takes place at the start of each month to keep the weights in “Large Growth” at 20%, in “Small Value” at 25%, in “Small Growth” at 35% and in “Large Value” at 20%. The tactical asset allocation has increased the correlation of “Large Value” from 0.18 to 0.52⁵. Because of the assumption of constant weights, we can derive the volatility of the contributions from the volatility of return and the weights, which gives for “Large Value”

$$\sigma(\underline{w}_i \underline{r}_i) = \underline{w}_i \times \sigma(\underline{r}_i) = 0.25 \times 9.59 = 2.3975\%$$

$$\sigma(w_i r_i) = w_i \times \sigma(r_i) = 0.20 \times 9.59 = 1.918\%$$

The allocation effect associated to “Large value” is

$$\sum_{i=1}^N \underbrace{\rho(\underline{w}_i \underline{r}_i, r_B) \times (\sigma(w_i r_i) - \sigma(\underline{w}_i \underline{r}_i))}_{\text{Allocation effect}} = 0.18 \times (1.918 - 2.3975) = -0.09\%$$

This is obviously the same result as in Table -2- but, as the model uses contributions to return instead of returns, it allows for trading during the period.

The second term gives the impact of a change in the correlations. It is equal to the difference between the correlation of the fund minus the benchmarks correlation, times the cross contribution.

This suggests that the manager is active in allocation and passive in stock picking. For “Large Value”, the calculation gives the following diversification effect

$$\sum_{i=1}^N \underbrace{\rho(w_i r_i, r_p) - \rho(\underline{w}_i \underline{r}_i, r_B)}_{\text{Diversification effect}} \times \sigma(w_i r_i) = (0.52 - 0.18) \times 1.918 = 0.65\%$$

The effect associated to stock picking is measured as the difference between the volatility of the portfolio contributions to return and the cross contribution, times the portfolio’s correlation. The selection effect caused by “Large Value” is

$$\sum_{i=1}^N \rho(w_i r_i, r_p) \times (\sigma(w_i r_i) - \sigma(\underline{w}_i \underline{r}_i)) = 0.52 \times (0.2 \times 9.31 - 0.2 \times 9.59) = -0.03\%$$

⁵ As we assume constant weights, the correlation of return contributions is equal to the correlation of returns. Also, the volatility of contributions to return is equal to the volatility of return times its weight.

The calculation uses the actual investment in “Large Value” (20%), the volatility that results from selectivity (9.31%) and the volatility of the passive strategy (9.59%). Once again, the results are similar to Table -2-.

To generalize the previous example, we suppose that, during an 18-months period, the manager has been active in each class. The weights that result from the active management decisions are illustrated in Figure 2.

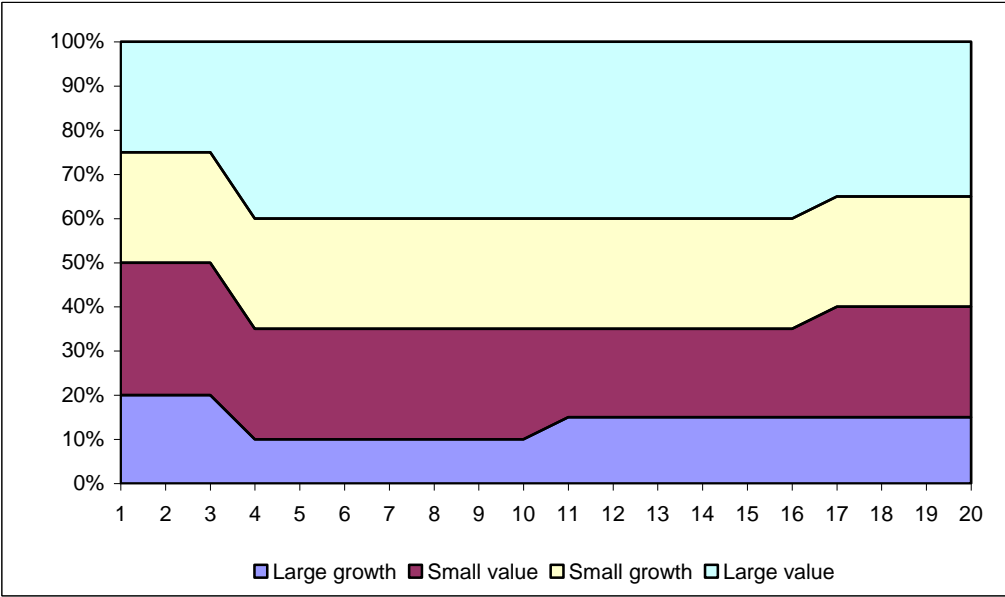


Figure -2- Investment in each asset class over an 18-months period

Table -3- shows that the managed portfolio has delivered a return equals to 12.94% compared to 10.27% for the benchmark or the strategic allocation. Active management (tactical allocation and selection) has also contributed to a higher volatility, from 5.88% for the benchmark to 8.61% for the portfolio. Changes in correlations and volatilities explain the higher risk.

	Fund %	Bench %	Fund Ret	Bench Ret	Vol P	Vol B	Correl P	Correl B	Correl TP	Contrib P	Contrib B	Contrib TP
Large growth	14%	25%	6.43%	9.25%	12.26%	9.15%	0.47	0.43	0.37	1.74%	2.29%	1.42%
Small value	24%	25%	10.55%	8.25%	17.40%	11.50%	0.48	0.58	0.59	4.30%	2.87%	2.72%
Small growth	25%	25%	13.48%	12.94%	21.36%	15.61%	0.73	0.72	0.72	5.34%	3.90%	3.90%
Large value	37%	25%	9.03%	7.37%	9.31%	9.59%	0.53	0.18	0.18	3.49%	2.40%	3.72%
Total			12.94%	10.27%	8.61%	5.88%				8.61%	5.88%	5.61%

Table -3- Return and risk by asset class (P = Portfolio, B = Benchmark and TP = Tactical Portfolio)

As a consequence of tactical allocation decisions, “Large Value” is significantly overweighted, therefore the correlation with the managed portfolio increased to 0.53 from 0.18. Basic finance theory tells us that higher correlation increases the risk of the portfolio, thus we expect a positive allocation associated to that class. Whereas overweighting “Large Value” has changed the level of risk, selection has reduced the risk of this asset class, from $\sigma(r_j) = 9.59\%$ to $\sigma(r_i) = 9.31\%$. Equation 3 is used to measure the effects caused by the changes of correlation, the impact of allocation and selection decisions. The results are presented in table -4-, it shows that the main contributor to a higher risk is “Large Value” for as much as 1.43% (allocation + diversification + selection), or 50% of the total risk’s increase.

	Fund %	Bench %	Fund Ret	Bench Ret	Vol P	Vol B	Allocation	Diversific	Selection
Large growth	14%	25%	6.43%	9.25%	12.26%	9.15%	-0.37%	0.06%	0.15%
Small value	24%	25%	10.55%	8.25%	17.40%	11.50%	-0.09%	-0.28%	0.75%
Small growth	25%	25%	13.48%	12.94%	21.36%	15.61%	0.00%	0.03%	1.04%
Large value	37%	25%	9.03%	7.37%	9.31%	9.59%	0.23%	1.32%	-0.12%
Total			12.94%	10.27%	8.61%	5.88%	-0.22%	1.12%	1.83%

Table -4- Risk attribution results.

The higher correlation resulting from overweighting “Large Value” increases the risk by 1.32% = $(0.53-0.18) \times 3.72\%$ while the larger amount of risk allocated to “Large Value” explains 0.23% = $0.18 \times (3.72\% - 2.40\%)$. Finally, as the manager selects some specific stocks, the risk of the investments in “Large Value” is actually 9.31% on an annual basis. This lower volatility reduces the total risk of the portfolio by $-0.12\% = 0.53 \times (3.49 - 3.72)$. The same calculations are done for each asset class and the contributions of all decisions for all asset classes are detailed in Table -4-.

Similar results are obtained from equation 4 a and b. Table -5- illustrates the different effects.

	Fund %	Bench %	Fund Ret	Bench Ret	Vol P	Vol B	Tactical Portfolio		Active Portfolio	
							Risk Allocation	Diversific	Risk Selection	Diversific
Large growth	14%	25%	6.43%	9.25%	12.26%	9.15%	-0.37%	-0.38%	0.05%	0.54%
Small value	24%	25%	10.55%	8.25%	17.40%	11.50%	-0.09%	-0.10%	0.86%	-0.29%
Small growth	25%	25%	13.48%	12.94%	21.36%	15.61%	0.00%	-0.30%	0.92%	0.44%
Large value	37%	25%	9.03%	7.37%	9.31%	9.59%	0.23%	1.03%	-0.10%	0.27%
Total			12.94%	10.27%	8.61%	5.88%	-0.22%	0.25%	1.73%	0.97%

Table -5- Risk attribution results.

The results in Table -5- are more intuitive as they show the changes in the risk profile related to the tactical decisions and the selection decisions. Setting a tactical portfolio changes the correlations and the amount of risk allocated to each asset class, which modifies the risk profile by 1.26% for the class “Large Value”. The selection decisions have an effect on the volatility of each asset class but also on the correlations. For this reason, we observe that selectivity has a positive combine effect of 0.17% on the volatility of “Large Value”. The analysis based on successive portfolios, tactical and then active, shows that the higher risk comes essentially from selectivity. Stock picking contributes to a total increase of 2.70% and tactical allocation to a small 0.03% increase. The total effect is 2.73% which is exactly the augmentation of risk (8.61% - 5.88%).

Conclusion

In this article, we have presented a model for risk attribution that explains the difference of volatility between a portfolio and its benchmark. The model is suitable for portfolios that use a strategic asset allocation as a benchmark and that deviate from the strategic allocation by deciding on tactical allocation and stock picking. We have proposed in this article two versions of the model; the first one identifies the changes in the risk profile by diversification, allocation and selection decisions; the second version measures the variations that originate from setting a tactical portfolio and then from stock picking. This second version is similar to a successive portfolio methodology. The model proposed in this article should contribute to a better understanding of the sources of performance of funds that are managed either against a benchmark or that are constrained by a specific strategic asset allocation.

Bibliography

Bertrand Philippe. 2008. “ Risk Attribution and Portfolio Optimizations under Tracking-error Constraints”. *The Journal of Performance Measurement*, 53-66, Vol 13, Fall 2008.

Brinson, G. P., L.R. Hood, and G.L. Beebower. 1986. “Determinants of Portfolio Performance”. *Financial Analysts Journal*. (July-August)

Fama, Eugene. 1972. “Components of Investment Performance”. *Journal of Finance*, vol. 17, no. 3 (June): 551-567.

Fong, Gifford, and Oldrich A. Vasicek. 1997. “A Multidimensional Framework for Risk Analysis”. *Financial Analysts Journal*, vol. 7, no. 8 (July/August): 51-57.

Goodwin, Thomas H. 1998, “The Information Ratio.” *Financial Analysts Journal*. (July/August), 34-43.

Grégoire, Philippe H. and Hervé Van Oppens: Risk Attribution, *The Journal of Performance Measurement*, 67-77, Vol 11, Fall 2006.

Grégoire, Philippe H.: Risk Attribution (pp. 309-329) in *Advanced Portfolio Attribution Analysis*; RiskBooks 2007 edited by Carl Bacon.

Karnosky, D.S., and B.D. Singer. 1994. “Global Asset Management and Performance Attribution”. *The Research Foundation of the Institute of Chartered Financial Analysts*.

Menchero, José and Hu Junmin. 2006. “Portfolio Risk Attribution”. *The Journal of Performance Measurement*.

Xiang, George. 2006. “Risk decomposition and its Use in Portfolio Analysis”. *The Journal of Performance Measurement*. Winter 2005/2006, pp. 26-32.

Appendix I: Data

Allocation

	Portfolio				Benchmark			
	Large cap Growth	Large cap Value	Small cap Growth	Small cap Value	Large cap Growth	Large cap Value	Small cap Growth	Small cap Value
Jan	25%	25%	25%	25%	25%	25%	25%	25%
Feb	26%	23%	26%	25%	25%	25%	25%	25%
Mar	27%	21%	27%	25%	25%	25%	25%	25%
Apr	28%	19%	28%	25%	25%	25%	25%	25%
May	29%	17%	29%	25%	25%	25%	25%	25%
Jun	30%	18%	27%	25%	25%	25%	25%	25%
Jul	31%	19%	25%	25%	25%	25%	25%	25%
Aug	29%	23%	23%	25%	25%	25%	25%	25%
Sep	27%	27%	21%	25%	25%	25%	25%	25%
Oct	25%	31%	19%	25%	25%	25%	25%	25%
Nov	23%	32%	18%	27%	25%	25%	25%	25%
Dec	21%	33%	17%	29%	25%	25%	25%	25%
Jan	19%	34%	16%	31%	25%	25%	25%	25%
Feb	21%	37%	13%	29%	25%	25%	25%	25%
Mar	23%	38%	12%	27%	25%	25%	25%	25%
Apr	25%	39%	11%	25%	25%	25%	25%	25%
May	27%	37%	13%	23%	25%	25%	25%	25%
Jun	29%	35%	15%	21%	25%	25%	25%	25%
Jul	31%	30%	17%	22%	25%	25%	25%	25%
Aug	33%	25%	19%	23%	25%	25%	25%	25%

Prices

	PORTFOLIO				BENCHMARK			
	Large cap Growth	Large cap Value	Small cap Growth	Small cap Value	Large cap Growth	Large cap Value	Small cap Growth	Small cap Value
Jan	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
Feb	98.6	98.8	95.9	90.9	95.6	97.8	100.9	102.0
Mar	95.5	96.4	99.6	90.5	97.0	96.7	95.5	101.2
Apr	100.9	96.1	109.0	98.2	96.3	99.0	105.0	101.4
May	106.5	101.0	104.6	97.4	97.3	102.9	103.0	98.4
Jun	101.5	104.4	110.8	103.5	96.5	106.2	104.2	102.5
Jul	97.5	107.8	109.8	99.7	96.8	105.0	99.1	106.4
Aug	101.1	107.9	109.6	105.3	100.3	106.4	108.5	107.1
Sep	103.2	108.1	107.5	119.5	100.5	106.3	103.9	108.2
Oct	102.6	105.3	105.1	115.1	102.5	112.8	103.5	105.9
Nov	107.4	105.7	105.8	129.3	103.9	112.5	104.7	110.3
Dec	106.7	107.9	98.5	142.4	101.2	117.2	105.8	116.2
Jan	105.6	110.6	92.5	141.1	97.8	111.9	102.9	116.6
Feb	104.2	108.6	91.4	134.9	99.2	117.8	100.9	115.8
Mar	100.2	106.8	92.7	127.7	104.0	119.1	103.2	114.0
Apr	100.2	105.6	99.8	145.1	104.4	117.3	103.7	118.4
May	99.2	109.4	101.8	140.0	110.7	115.0	106.2	115.7
Jun	105.1	111.0	110.5	149.5	109.9	118.1	110.7	112.1
Jul	102.4	114.9	106.9	144.3	111.5	110.5	118.2	108.4
Aug	106.4	109.0	109.3	138.1	109.2	108.3	112.9	107.4