Performance Attribution for Portfolios that Trade Futures Contracts

In this article, we propose an attribution model for a leveraged or hedged portfolio that is in line with Brinson et al. *The model takes into account the effect of futures in allocation, displays a selection effect that is due solely to securities and not futures, measures the leverage effect, and, finally, isolates the return that is caused by an imperfect correlation between futures and the underlying asset class.*

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INTRODUCTION

Futures are often used to change the actual asset allocation of an actively managed portfolio. If the manager is bearish on a specific asset class, he may want to sell some of his holdings or, as an alternative, he can sell futures that are correlated with this asset class. In terms of performance attribution, the use of futures causes a new allocation that should be reflected in the allocation effect. This means that what matters for allocation is the exposure to an asset class rather than portfolio holdings. Another point of interest is the way the return of an asset class is calculated when futures are used to change exposure. To illustrate this, let us assume that equity represents 40% of a portfolio that's worth \$100 million. Return is 5%, or in other words, profits of equity over the period are equal to \$2 million. The manager decides to reduce half of his exposure to equity by selling futures that are perfectly correlated with his equity holding. The consequences of this decision are an exposure of 20% to equity, a profit (equity) of \$2 million and a loss (futures) of \$1 million (as we have assumed a perfect correlation). The questions that arise are: how to calculate the return of the equity bucket? What is the return of the futures? What are the weights that should be used to calculate the allocation effect? Do futures cause a selection effect? How do we measure the impact of the leverage effect? How do we take into account the imperfect correlation between holdings and futures? Calculating attribution with futures requires that we propose acceptable answers to these questions.

In this article, we propose an attribution model for leveraged or hedged portfolio that is in line with Brinson *et al.* The model takes into account the effect of futures in allocation, displays a selection effect that is due solely to securities and not futures, measures the leverage effect, and, finally, isolates the return that is caused by an imperfect correlation between futures and the underlying asset class. The article is written following the same logic and objective that we used during our research, meaning we establish a model of attribution including futures where attribution effects are intuitive, similar to what Brinson *et al.* established, and, of course, do sum to the excess return. As the model is additive, linking and chaining algorithms are used to calculate the different effects over a longer period.

In the first section, we cover returns calculation with futures,[1] and we show how to express the total return as a weighted sum of returns on exposure or, in other words, how to calculate the portfolio return as a sum of contributions to return. The attribution model and the different effects, allocation, selection, leverage and basis, are explained in section 2. Section 3 illustrates the model and proposes an interpretation of the results.

RETURNS CALCULATION

A necessary condition that shows how attribution results sum up to the excess return is that both the portfolio and the benchmark return are expressed as the sum of contributions to the return. While the weights are observable for indices, they have to be calculated for portfolios. Let us assume that the portfolio remains stable over the period, *i.e.,* no trades or cash contributions / withdrawals. In such a case, the total return is equal to the weighted sum of individual returns, weights being equal to the asset's initial value divided by the portfolio total value. Once futures are used, we map these instruments into a corresponding exposure in the underlying asset and an equivalent cash position. Doing so ensures that the total return is equal to a weighted sum of returns where weights and returns are expressed in terms of exposures rather than holdings.

As there are no transactions or cash movements, the total return is equal to the P&L divided by the initial portfolio value.

$$\boldsymbol{R_P} = \frac{P\&L}{V_0}$$

We can insert in this equation the capital invested in the portfolio, the underlying futures exposure, and the cash equivalent position that maps the futures' exposure into this equation.

$$R_P = \frac{V_{Inv}}{V_0} \times \frac{P\&L_{Inv}}{V_{Inv}} + \frac{V_{Fut}}{V_0} \times \frac{P\&L_{Fut}}{V_{Fut}} + \frac{V_{Map}}{V_0} \times \frac{P\&L_{Map}}{V_{Map}}$$

We can interpret this equation as the sum of contributions to return, the contributions of futures being decomposed into a first term that depends on the underlying, and second term that is associated with the cash equivalent position. Let us illustrate these calculations with a portfolio that partially hedges equity holdings.

$$R_P = \frac{264{,}600}{1{,}200{,}000} = 5.38\%$$

And,

$$5.38\% = \frac{400}{1{,}200} \times \frac{28}{400} + \frac{700}{1{,}200} \times \frac{56}{700} + \frac{-340}{1{,}200} \times \frac{-20.4}{-340} + \frac{340}{1{,}200} \times \frac{0}{340} + \frac{100}{1{,}200} \times \frac{1}{100}$$

To calculate the contributions to return, we have assumed that the return of the cash equivalent holding is equal to 0. As the theoretical value of futures is equal to the spot price plus the cost of carry, we can use the theoretical pricing formula to calculate the cost of carry on the cash equivalent position.

To illustrate this, let us assume a short term rate of 2% and a holding period of three months.

Table 1

	V_0	V_1	P&L	Return	Contribution
Bond	400,000	428,000	28,000	7.00%	2.33%
Equity (securities)	700,000	756,000	56,000	8.00%	4.67%
Equity (Futures)	-340,000	-360,400	-20,400	6.00%	-1.70%
Map (Futures)	340,000	340,000			0.00%
Cash	100,000	101,000	1,000	1.00%	0.08%
Total	**1,200,000**	**1,264,600**	**64,600**	**5.38%**	**5.38%**

Table 2

	V_0	V_1	P&L	Return	Contribution
Bond	400,000	428,000	28,000	7.00%	2.33%
Equity (securities)	700,000	756,000	56,000	8.00%	4.67%
Equity (Futures)	-340,000	-362,100	-22,100	6.50%	-1.84%
Map (Futures)	340,000	341,700	1,700	0.50%	0.14%
Cash	100,000	101,000	1,000	1.00%	0.08%
Total	**1,200,000**	**1,264,600**	**64,600**	**5.38%**	**5.38%**

Contributions to total return are shown in Table 2.

Theoretically, selling futures is equivalent to sell short the underlying and investing the proceeds at the risk-free rate. Interest earned (or paid) on this risk-free investment represent the cost of carry for futures on equities that do not distribute dividends.

For a three-month period, the cost of carry is then

$$Carry = \frac{2\% \times 340{,}00 \times 0.25}{340{,}000} = 0.5\%$$

We then remove the cost of carry of futures P&L to obtain the return that is theoretically associated with the underlying securities.

$$Return_{Futures} = \frac{-20{,}400 - 1{,}700}{340{,}000} = 6.5\%$$

As contributions to return are additive, the contribution to return of the equity asset class after hedging is equal to 4.67% − 1.84% = 2.83 percent. In the next section, we will use these contributions to calculate the different effects for an attribution model that reflects a top-down management process, *i.e.,* allocation then selection.

ATTRIBUTION MODEL WITH FUTURES

In this section, we propose a model that explains the excess return over the benchmark as the sum of four effects:

- an allocation effect that measures the impact of active exposure on the excess return,
- a selection effect that identifies the active return associated with cash securities,
- a leverage effect that shows the impact of using derivatives,
- and a basis effect that accounts for the imperfect correlation between the futures and the underlying prices.

Allocation effect

Using futures not only changes the portfolio allocation but it also creates a leverage effect. We expect an attri-

Table 3

	V_0	Exposure	Index weights	Portfolio return	Index return
Bond	400,000	33.33%	40.00%	7.00%	4.00%
Equity (securities)	700,000	58.33%	40.00%	8.00%	6.00%
Equity (Futures)	-340,000	-28.33%	0.00%	6.50%	-
Map (Futures)	340,000	28.33%	0.00%	0.50%	-
Cash	100,000	8.33%	20.00%	1.00%	1.00%
Total	**1,200,000**	**100%**	**100%**	**5.38%**	**4.20%**

Asset Class	Exposure	Index weights	Weight difference	Index return	Allocation effect
Bond	33.33%	40.00%	-6.67%	4.00%	0.01%
Equity	30.00%	40.00%	-10.00%	6.00%	-0.18%
Cash	8.33%	20.00%	-11.67%	1.00%	0.37%
Total	**71.67%**	**100%**	**-28.33%**	**4.20%**	**0.21%**

Table 4

bution model to isolate the pure allocation effect from others effects. Allocation must be proportional to the difference between the portfolio exposure in an asset class minus the benchmark weight. In the example presented in the previous section, equity exposure is (700-340)/1,200=30 percent. Table 3 takes the same example but adds the index composition and return.

The allocation effect measures the impact of investing a budget, equal to the difference between the portfolio exposure and the benchmark, in an investment vehicle that exactly tracks the index. In our example, the manager investment universe is made of three asset classes: Bond, Equity, and Cash. The corresponding budgets are -6.67% = 33.33% - 40%, -10% = 30% - 40% and -11.67% = 8.33% - 20 percent. These budgets do not take into account the cash equivalent or mapping position. Table 4 shows the allocation effects that correspond to the net exposure in each asset class.

The allocation effect in bonds is tiny as the bond index return is close to the global index return (4% vs 4.2%). As the net equity exposure is lower than that of the index (-10% underweight), the allocation effect associated to equities is negative (-0.18%) as the portfolio is underweighted in a strategy which index return is above that of the global index. On the other hand, the allocation effect in cash is equal to 0.37% = (8.33% - 20.00%) ´ (1.00% - 4.20%). The portfolio manager is logically rewarded as he decided to underweight the cash bucket in a scenario where the global index has returned more than the risk-free return. These results are thus in line with our intuition.

However, we can argue that, in the same way equity allocation is impacted by the short futures position, the equivalent cash position should impact the cash allocation effect. Although this is theoretically correct, we favor a methodology that isolates the effect of leverage associated with futures. The resulting exposure on each asset class instead of including it in the allocation effect. This leverage effect will be explained later in this article.

As illustrated in the previous example, we will define the allocation effect as the difference between the portfolio exposure and the benchmark weights times the excess return of the asset class over the benchmark. By using the index return (that represents the asset class) for asset classes, including futures, indirectly suppose that futures returns are exactly equal to that of the index. This is unlikely because usually futures returns are slightly different from returns of the corresponding asset class. We consider that this potential return difference is not an allocation choice and should be isolated as a separate effect, the basis effect, which will be discussed later on.

This example leads us to write the allocation effect as[2]

$$A_i = \left(w_i^{exp} - \underline{w_i}\right) \times \left(\underline{R_i} - \underline{R}\right)$$

A_i is the allocation effect for asset class i.

w_i^{exp} is the asset class exposure. The cash equivalent exposure is not included in this term.

$\underline{w_i}$, $\underline{R_i}$, and $\underline{R}$ are benchmark weight, index return, and return of the blended benchmark.

We note that the sum of the asset class exposures is not equal to 1, as we do not include the cash equivalent exposure. This suggests that the use of the relative return, ($\underline{R}$i - $\underline{R}$), is neutral only if we include the equivalent cash

Table 5						
	V_0	Exposure	Index weights	Portfolio return	Index return	Selection effect
Bond	400000.00	33.33%	40.00%	7.00%	4.00%	1.00%
Equity (securities)	700000.00	58.33%	40.00%	8.00%	6.00%	1.17%
Equity (Futures)	-340000.00	-28.33%	0.00%	6.50%	-	-
Map (Futures)	340000.00	28.33%	0.00%	0.50%	-	-
Cash	100000.00	8.33%	20.00%	1.00%	1.00%	0.00%
Total	1200000.00	100.00%	100.00%	5.38%	4.20%	2.17%

exposure. This will be done when we discuss the leverage term.

Selection effect

This effect measures the excess return due to stock picking. Most of the time futures returns are linked to the return of a basket of securities or of an index. Futures are thus overall not bought/sold on selection purpose and should not impact the selection effect of the portfolio. Another intuitive requirement is that the selection effect is proportional to the "cash" budget effectively invested and not to the exposure. Intuitively, the selection effect associated with a particular asset class *i* is equal to the effectively invested budget times excess return, *i.e.,*

$$S_i = w_i^{Inv} \times \left(R_i^{Inv} - \underline{R_i}\right)$$

S_i is the selection effect for asset class *i*.

w_i^{Inv} is the weight invested directly (*i.e.,* not with futures) in asset class *i*. Weights associated with both the futures and to the cash equivalent do not enter into the calculation of the selection effect.

R_i^{Inv} and R_i are the return of the securities and the return of the index that represents the asset class *i*.

Let us consider the same portfolio that we have previously detailed. Table 5 details the computation of the selection effect.

Bonds have delivered a return of 7% higher than the index. Selection effect is positive and equal to 1.00% = 33.33% ´ (7.00% - 4.00%). For stocks, we only consider the budget that is effectively invested, *i.e.,* without futures exposure. Stock selection is therefore 1.17% = 58.33% ´ (8.00% - 6.00%). As cash return is equivalent in both the portfolio and the index, the selection effect is null for the cash bucket.

The selection effect computed here is exactly the selection effect of a portfolio that does not trade futures. This is consistent with our hypothesis, which assumes that futures are used to change asset allocation and that they do not account for selection.

Note also that the first two terms that we have just presented are the same as those obtained under the Brinson *et al*. model. The only difference stands in the use of exposure instead of weights for the allocation. The part of the excess return that remains unexplained by these terms, allocation and selection, will now be decomposed into two additional terms, the leverage and the basis effects.

Leverage effect

Futures create leverage because buying futures is equivalent to borrowing cash and using the proceeds to buy a basket of securities. In the same way, selling futures is mapped into a short position in the asset class and an investment in a cash position. In both cases, trading futures generates an equivalent cash position equal to the market value of the underlying securities. To illustrate the leverage effect, let us take a portfolio with two positions. The first one is an investment in an ETF that tracks the stock market, and the second one a long position in a future on the same ETF. Let us assume that the ETF return is 10% and that the portfolio initial value is 500. The underlying position of the future is 300. Table 6 summarizes the portfolio's setup.

Table 6			
		Value	Return
Buy	ETF	500	10%
Long	Futures	300	10%

To calculate the leverage effect, we refer to the well-known Modigliani and Miller (1958) Proposition 2 that establishes a relationship between the return on equity (ROE) and the return on assets (ROA). We assume that the total cash investment in the portfolio is similar to equity (E) and that the total return, which includes derivatives (D), is the ROE. The total return on investment in securities, derivatives excluded, is the ROA. Applying these rules to the portfolio presented above gives a ROA of 10% (50/500) and a ROE of 16% ((50+30)/500).

The relation between ROE and ROA states that ROE is equal to ROA plus leverage.

$$ROE = ROA + \frac{D}{E} \times (ROA - r_f)$$

ROA is the return on effective investment in the asset class, *i.e.,* returns on holding securities. ROE is the return of the portfolio that contains securities and derivatives. Rewriting this relationship to define the difference ROE – ROA, we get a formula to compute leverage. Temporarily, we accept that the cost of carry, r_f, is 0. Therefore, looking at exposures and returns that are given in Table 6 and applying the Modigliani and Miller Proposition 2, we calculate the leverage effect.

$$16.0\% - 10\% = 6\% = \frac{300}{500} \times (10.0\% - 0.0\%)$$

Since we do not compare the portfolio to a benchmark in this example, we have implicitly assumed that the equivalent amount of debt (300) is invested in the ETF. When we calculate the leverage effect in a portfolio measured against a benchmark, we suppose, in order to measure a leverage effect as pure as possible, that the equivalent amount of debt is invested passively in the benchmark. We can now rewrite the leverage effect in our attribution model as

$$L_i = \frac{-V_i^{map}}{V_0} \times \left(\underline{R} - R_i^{map}\right) \quad \text{or}$$

$$L_i = w_i^{map} \times \left(R_i^{map} - \underline{R}\right)$$

L_i is the leverage effect.

$-V_i^{map}$ is the equivalent debt. The negative sign comes from the fact that the equivalent amount of cash is negative for a long futures position. V_0 is the value of the initial portfolio.

$\underline{R}$ is the benchmark return.

w_i^{map} is the exposure associated to the cash equivalent position.

R_i^{map} is the cost of carry.

This formula also fits with our intuition. When leverage is used in a context where the benchmark return is above that of cash $\underline{R} > R_i^{map}$, the leverage effect is positive, so this was a good decision. On the other hand, the effect becomes negative when the global benchmark return is lower than cash $\underline{R} > R_i^{map}$, which represents a bad scenario to leverage. Note that these relationships are inverted when the leverage is negative. For example, a short future position, in a context of cash return lower than the benchmark return, will induce a negative leverage effect.

In Table 7, we calculate the leverage effect for the portfolio that we have used in Table 1.

The leverage effect is shown on the same line as the futures which is -1.05% = 28.33%´(0.5% - 4.2%), because we still assume that the cost of carry is 0. In this example, the portfolio is short 340,000, meaning the cash

Table 7

	V_0	Exposure	Index weights	Portfolio return	Index return	Leverage effect
Bond	400000.00	33.33%	40.00%	7.00%	4.00%	-
Equity (securities)	700000.00	58.33%	40.00%	8.00%	6.00%	-
Equity (Futures)	-340000.00	-28.33%	0.00%	6.50%	-	-1.05%
Map (Futures)	340000.00	28.33%	0.00%	0.50%	-	-
Cash	100000.00	8.33%	20.00%	1.00%	1.00%	-
Total	1200000.00	100.00%	100.00%	5.38%	4.20%	-1.05%

equivalent is similar to an investment. The leverage effect is negative because the manager is short futures, thus long a cash equivalent position while the global index is positive. The leverage effect can therefore be understood as the contribution to the excess return of the cash equivalent position.

Basis effect

Futures contracts trade at prices that are usually different than spot prices because the price of futures includes the cost of carry of the underlying assets. On top of this, futures prices react more quickly to expectation changes than spot prices. This means that futures returns are not perfectly correlated with the underlying asset. We also have to consider that futures contracts may not be available for some asset classes, which means that a manager could decide to trade futures that have an imperfect correlation with the portfolio asset class. When futures exist, the difference between futures and spot price is called the basis, and it is a function of the cost of leverage and market uncertainty. In our attribution context, as we have already included the cost (or benefit) of leverage in the leverage effect term, the basis effect will measure the excess return that results from imperfect correlation. This term is proportional to the difference of return between futures and index representing the asset class. The basis effect is given by

$$B_i = w_i^{Fut} \times \left(R_i^{Fut} - \underline{R_i}\right)$$

B_i is the basis effect.

w_i^{Fut} is the exposure to the asset class i relative to the futures.

R_i^{Fut} and $\underline{R_i}$ are the return of the futures and the index associated to the asset class.

Table 8 computes the basis effect for the portfolio of Table 1.

In Table 9, we summarize all effects (allocation, selection, leverage and basis) and show that their sum is exactly equal to the excess return. In the next section, we will demonstrate that this is always the case as long as

Table 8

	V_0	Exposure	Index weights	Portfolio return	Index return	Basis effect
Bond	400000.00	33.33%	40.00%	7.00%	4.00%	-
Equity (securities)	700000.00	58.33%	40.00%	8.00%	6.00%	-
Equity (Futures)	-340000.00	-28.33%	0.00%	6.50%	-	-0.14%
Map (Futures)	340000.00	28.33%	0.00%	0.50%	-	-
Cash	100000.00	8.33%	20.00%	1.00%	1.00%	-
Total	1200000.00	100.00%	100.00%	5.38%	4.20%	-0.14%

Table 9

	Exposure	Index weights	Portfolio return	Index return	Allocation effect	Selection effect	Leverage effect	Basis effect	Total effect
Bond	33.33%	40.00%	7.00%	4.00%	0.01%	1.00%	-	-	1.01%
Equity (sec.)	58.33%	40.00%	8.00%	6.00%	-0.18%	1.17%	-	-	0.99%
Equity (Futures)	-28.33%	0.00%	6.50%	-			-1.05%	-0.14%	-1.19%
Map (Future)	28.33%	0.00%	0.50%	-			-	-	-
Cash	8.33%	20.00%	1.00%	1.00%	0.37%	0.00%	-	-	0.37%
Total	100.00%	100.00%	5.38%	4.20%	0.21%	2.17%	-1.05%	-0.14%	1.18%

we are able to express the total return as a sum of contributions. As the model is additive, we can use chaining (linking) algorithm to ensure that the sum of the multi-periodic effects are equal to the excess return.

We verify in this example that the sum of the four effects is exactly equal to the excess return.

This example shows that it is possible to explain the excess return of a portfolio that trades futures to change the asset allocation. The different terms give numbers in line with intuition and highlight the specific features of derivatives, leverage, and price basis.

Attribution Model when Futures are Used to Change the Asset Allocation

For attribution purposes, portfolio and benchmark returns must be expressed as a weighted sum. Over a single period, we assume that weights account for trading and that return calculations are consistent with these weights. The portfolio return is given by

$$R_P = \sum_{i=1}^{N} w_i \times R_i$$

When futures are traded in the portfolio, they are mapped into two legs: the exposure in the underlying asset and the cash equivalent. Then, the portfolio return is

$$R_P = \sum_{i=1}^{N} w_i^{Inv} \times R_i^{Inv} + \sum_{i=1}^{K} w_i^{Fut} \times R_i^{Fut} + \sum_{i=1}^{K} w_i^{Map} \times R_i^{Map}$$

w_i^{Inv} and R_i^{Inv} are the weight and return associated to asset holdings.

w_i^{Fut} and R_i^{Fut} are the exposure and return associated to future contracts.

w_i^{Map} and w_i^{Map} are the exposure and return associated to the equivalent cash position.

And the benchmark return is

$$\underline{R} = \sum_{i=1}^{N} \underline{w_i} \times \underline{R_i}$$

For each term of the portfolio return, we apply Brinson decomposition to identify allocation and selection effect.

$$R_P - \underline{R} = \sum_{i=1}^{N} \left(w_i^{Inv} + w_i^{Fut} + w_i^{Map} - \underline{w_i}\right) \times \left(\underline{R_i} - \underline{R}\right) + \sum_{i=1}^{N} w_i^{Inv} \times \left(R_i^{Inv} - \underline{R_i}\right) \sum_{i=1}^{N} w_i^{Fut} \times \left(R_i^{Fut} - \underline{R_i}\right) + \sum_{i=1}^{N} w_i^{Map} \times \left(R_i^{Map} - \underline{R_i}\right)$$

We rearrange the first term of the equation to show allocation with futures and to remove the term in cash equivalent.

$$R_P - \underline{R} = \sum_{i=1}^{N} \left(\underbrace{\left[w_i^{Inv} + w_i^{Fut}\right]}_{Exposure} - \underline{w_i} \right) \times \left(\underline{R_i} - \underline{R}\right) + \sum_{i=1}^{N} w_i^{Inv} \times \left(R_i^{Inv} - \underline{R_i}\right) + \sum_{i=1}^{N} w_i^{Map} \times \left(\underline{R_i} - \underline{R}\right) + \sum_{i=1}^{N} w_i^{Map} \times \left(R_i^{Map} - \underline{R_i}\right) + \sum_{i=1}^{N} w_i^{Fut} \times \left(R_i^{Fut} - \underline{R_i}\right)$$

Finally, we can reorganize the term to the cash equivalent to obtain the leverage effect, and we have thus demonstrated our attribution model that was established from an intuitive approach in the previous section.

$$R_P - \underline{R} = A_i + S_i + L_i + B_i$$

Where

$$A_i = \sum_{i=1}^{N} \left(\underbrace{\left[w_i^{Inv} + w_i^{Fut}\right]}_{Exposure} - \underline{w_i} \right) \times \left(\underline{R_i} - \underline{R}\right)$$

$$S_i = \sum_{i=1}^{N} w_i^{Inv} \times \left(R_i^{Inv} - \underline{R_i}\right)$$

$$L_i = \sum_{i=1}^{N} w_i^{Map} \times \left(R_i^{Map} - \underline{R}\right)$$

$$B_i = \sum_{i=1}^{N} w_i^{Fut} \times \left(R_i^{Fut} - \underline{R_i}\right)$$

The first component, allocation, measures the impact of passively investing a budget equal to direct investment in assets plus exposure in futures contracts. Overweight or underweight against the benchmark is no longer expressed in terms of investment, but of exposure to an asset class. This term is similar to the standard allocation term in an attribution model. The only difference is that weights correspond to exposures instead of cash investments.

The second component, selection, concerns investments in cash securities. This term is exactly the same as in standard attribution. This is consistent with our assumption that limits the use of futures for allocation purposes.

Leverage, the third component, is specific to our model. To have a better understanding of this component, we should go back to the second proposition of Modigliani and Miller (1958) that demonstrates that the return on equity (ROE) of a firm is equal to the return on asset (ROA) plus leverage. Leverage is equal to the ratio debt/equity times the difference of the return on asset and the cost of the debt. Assuming that the total market value of the portfolio belongs to the holder, hence it is equity, the amount of debt is the cash equivalent of a long position in future contracts. The weight w_i^{Map} is equivalent to the leverage ratio in Modigliani and Miller. As the difference of return between the active strategy (portfolio return) and the passive strategy (benchmark return) is explained by the first two components, allocation and selection, we limit the impact of the leverage to the passive strategy. Explicitly, the benchmark return plays the role of the return on the asset.

The last term of our attribution model, the basis effect, measures the impact of imperfect correlation between the returns of futures contracts and those of the corresponding asset class. We have already analyzed effects of futures contracts on allocation and leverage, assuming that futures returns are perfectly correlated with the corresponding asset class and that their levels are given by the theoretical price. This is unusual because futures do not necessarily trade at their theoretical price, and futures contracts are not always available for each asset class.

CONCLUSION

The model that we have developed in this article adds insights to the existing versions of the Brinson models as it decomposes the effect of using futures to change the asset allocation into an allocation term, as in the Brinson model, and two new terms that are the leverage effect and the basis effect. Both terms give interesting information about the degree of leverage in the portfolio and the quality of the leverage /hedge investment vehicle chosen. Dealing with this model might lead in practice to some initial questions, particularly as it mixes weights in exposure and corresponding to cash investments, which is always tricky when dealing with futures. However we trust it can quickly become useful for its user, especially as all effects have been deducted from an intuitive process. This model can also be extended to a large class of derivatives, as these instruments can always be represented by exposure in different securities.

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ENDNOTES

[1] John C. Stannard, "Measuring Investment Returns of Portfolios Containing Futures and Options," *The Journal of Performance Measurement*, Fall 1996 - Volume 1 - Number 1.

[2] This is exactly the term of the Brinson *et al.* model (1991). The only difference is that the portfolio weights are exposures instead of holdings.